MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

TMT1181 – MATHEMATICAL TECHNIQUES II

(TC01 / TT01)

25 FEBRUARY 2017
2.30 p.m - 4.30 p.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **8** pages (inclusive of the cover page).
2. Answer **ALL** questions. The distribution of marks for each question is given.
3. The **Appendix** section begins from **page 6** onwards.
4. Please write all your answers in the Answer Booklet provided.
5. Please write the answers for each question on a new page of your Answer Booklet.

QUESTION 1: ORDINARY DIFFERENTIAL EQUATION (ODE)

(Total Marks: 12)

(a) A cup of coffee cools to room temperature according to the Newton's law of cooling and can be described by the following first-order ODE ,

$$\frac{dT}{dt} = k(T - m) \qquad T(0) = T_0$$

where, t is the time variable,
T is the temperature variable at time t,
k is some parameter value,
m is the room temperature (when coffee cools down),
and T_0 is the initial temperature (when coffee is hot).

Show that the **general solution** for the first-order ODE is $T = ce^{kt} + m$, where c is a constant value. (3 marks)

(b) Given the following **second-order ODE**,

$$y'' - 3y' + 2y = 4x^2$$

(i) Find the **homogenous solution** y_h for the ODE.

(ii) Find the **particular solution** y_p for the ODE.

(iii) Construct the **general solution** $y = y_h + y_p$.

(3 + 5 + 1 = 9 marks)

Continue...

QUESTION 2: LAPLACE TRANSFORM AND FOURIER SERIES

(Total Marks: 12)

(a) Use **Laplace Transform** to solve the following **first-order ODE with an initial-value at $t = 0$,**

$$y' + 2y = 4e^{-t} \qquad y(0) = 1$$

(7 marks)

(b) Given the following periodic function,

$$f(x) = \begin{cases} -x & -\pi < x < 0 \\ 0 & 0 \le x < \pi \end{cases} \qquad f(x) = f(x + 2\pi)$$

(i) **Sketch** the graph of $f(x)$ in the interval $-2\pi < x < 2\pi$ and indicate clearly the period of $f(x)$.

(ii) Find the **Fourier Series coefficient a_0** for this function.

(2 + 3 = 5 marks)

Continue...

QUESTION 3: INTERPOLATION AND POLYNOMIAL APPROXIMATION

(Total Marks: 13)

(a) Given the following function,

$$f(x) = x^3 + \cos(x)$$

(i) Find its **Maclaurin Polynomial**, $P_3(x)$.

(ii) Use the Maclaurin Polynomial from part (i) to **approximate $P_3(0.1)$**.

(5 + 1 = 6 marks)

(b) Given the following **divided-difference** table:

x_k	$y_k = f(x_k)$	1st Divided Difference	2nd Divided Difference	3rd Divided Difference	4th Divided Difference
$x_0 = 1$	$y_0 = 1$				
$x_1 = 2$	$y_1 = 8$				
$x_2 = 3$	$y_2 = 27$				
$x_3 = 5$	$y_3 = 125$				
$x_4 = 8$	$y_4 = 512$				

(i) Formulate the **Newton Interpolating Polynomial, $P_4(x)$** by completing the table.

(ii) Use $P_4(x)$ to **estimate the value for $x = 4$.**

(6 + 1 = 7 marks)

Continue...

QUESTION 4: NUMERICAL SOLUTIONS (Total Marks: 13)

(a) Given the following systems of linear equations,

$$x - 8y - 2z = 1$$
$$x + y + 5z = 4$$
$$3x - y + z = -2$$

(i) Rearrange the systems of linear equations so that they are **diagonally dominant**.

(ii) Find the values of x, y, z using the **Jacobi method with 2 iterations**. Begins your iteration with $x_0 = 0,\ y_0 = 0,\ z_0 = 0$. Round off your answers to 5 decimal places. $(3 + 4\frac{1}{2} = 7\frac{1}{2}$ marks)

(b) Given the following first-order ODE with an initial value at $x = 0$,

$$y' = \frac{x^3}{y^3} \qquad y(0) = 1$$

(i) Find the approximate solution using **Runge-Kutta 2nd Order Method in 3 iterations with a step size, h = 0.25.** Round off your answers to 6 decimal places.

(ii) If you want a more accurate solution for the ODE, what can you do with the value of the step size, h ?

$(4\frac{1}{2} + 1 = 5\frac{1}{2}$ marks)

End of Questions.

APPENDIX A: DERIVATIVE AND INTEGRAL TABLE

Derivative Table

1.	$\frac{d(kx^n)}{dx} = knx^{n-1}$	2.	$(u \pm v)' = u' \pm v'$	3.	$(u \cdot v)' = u'v + v'u$
4.	$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$	5.	$(u(v))' = u'(v) \cdot v'$	6.	$\frac{d(a^u)}{dx} = a^u \ln(a) \cdot u'$
7.	$\frac{d(e^u)}{dx} = e^u u'$	8.	$\frac{d(\log_u u)}{dx} = \frac{1}{u \ln(a)} \cdot u'$	9.	$\frac{d(\ln u)}{dx} = \frac{1}{u} \cdot u'$
10.	$\frac{d(\sin u)}{dx} = \cos u \cdot u'$	11.	$\frac{d(\cos u)}{dx} = -\sin u \cdot u'$	12.	$\frac{d(\tan u)}{dx} = \sec^2 u \cdot u'$
13.	$\frac{d(\cot u)}{dx} = -\csc^2 u \cdot u'$	14.	$\frac{d(\csc u)}{dx} = -\csc u \cot u \cdot u'$	15.	$\frac{d(\sec u)}{dx} = \sec u \tan u \cdot u'$

Integral Table

16.	$\int kx^n \, dx = k\frac{x^{n+1}}{n+1} + C; \quad n \neq -1$	17.	$\int \frac{1}{x} dx = \int x^{-1} \, dx = \ln \lvert x \rvert + C$
18.	$\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C$	19.	$\int a^{kx} \, dx = \frac{1}{k} \frac{a^{kx}}{\ln(a)} + C$
20.	$\int \frac{k}{ax+b} dx = \frac{k}{a} \ln \lvert ax+b \rvert + C$	21.	$\int u \, dv = u \cdot v - \int v \, du$
22.	$\int \sin x \, dx = -\cos x + C$	23.	$\int \cos x \, dx = \sin x + C$
24.	$\int \sec^2 x \, dx = \tan x + C$	25.	$\int \csc^2 x \, dx = -\cot x + C$
26.	$\int \sec x \, dx = \ln \lvert \sec x + \tan x \rvert + C$	27.	$\int \csc x \, dx = \ln \lvert \csc x - \cot x \rvert + C$
28.	$\int \tan x \, dx = \ln \lvert \sec x \rvert + C$	29.	$\int \cot x \, dx = \ln \lvert \sin x \rvert + C$
30.	$\int \frac{1}{1+x^2} du = \tan^{-1} x + C$	31.	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

APPENDIX B: LAPLACE TRANSFORM TABLE

Function, $f(t)$	Laplace Transform, $F(s)$
1	$\frac{1}{s}$ $s>0$
t	$\frac{1}{s^2}$ $s>0$
$e^{\alpha t}$	$\frac{1}{s-\alpha}$ $s>a$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$ $s>0$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$ $s>0$
t^n n is an integer	$\frac{n!}{s^{n+1}}$ $s>0$
$e^{\alpha t}t^n$ n is an integer	$\frac{n!}{(s-\alpha)^{n+1}}$ $s>a$
$t^n \cdot f(t)$ n is an integer	$(-1)^n \cdot \frac{d^n F(s)}{ds^n}$
$e^{\alpha t} f(t)$	$F(s-\alpha)$
$f(t)$	$F(s)$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$

APPENDIX C: TABLE OF FORMULAS

Roots for Quadratic Equation:

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Fourier Series: **Period,** $(-L, L)$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi}{L}x\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi}{L}x\,dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{L}x + b_n\sin\frac{n\pi}{L}x\right)$$

Lagrange Coefficient:

$$L_i(x) = \frac{(x-x_0)\cdot(x-x_1)\cdot(x-x_2)\cdots(x-x_{i-1})\cdot(x-x_{i-2})\cdots(x-x_n)}{(x_i-x_0)\cdot(x_i-x_1)\cdot(x_i-x_2)\cdots(x_i-x_{i-1})\cdot(x_i-x_{i-2})\cdots(x_i-x_n)} \qquad i = 0, 1, 2, \ldots, n$$

Taylor Series:

$$P_n(x) = \sum_{k=0}^{n}\frac{1}{k!}f^{(k)}(a)(x-a)^k$$

Maclaurin Series:

$$P_n(x) = \sum_{k=0}^{n}\frac{1}{k!}f^{(k)}(0)(x)^k$$

Euler Method:

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Runge-Kutta 2nd-Order Method:

$$k_1 = f(x_n, y_n)$$

$$k_2 = f(x_n + h, y_n + k_1 h)$$

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$